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The Symmetry and Harmonic Proportions of the Temples of Artemis and Zeus at Jerash; and The Origin of Numerals as used in the Enlargement of the South Theatre in Jerash

The symmetry in classical architecture

Vitruvius in Book III, chapter I, states: 'the planning of temples depends upon symmetry, and the method of this architects must diligently apprehend. It arises from proportion (which in Greek is called **analogia**). Proportion consists in taking a fixed module in each case, both from the parts of a building and for the whole, by which the method of symmetry is put into practice. For without symmetry and proportion no temple can have a regular plan; (that is it must have an exact proportion worked out after the fashion of the members of a finely shaped human body)'.

The Greek word **analogia** translated as 'proportion' does not convey the real sense of the Greek word; Nicomachus of Gerasa, in his *Introduction to Arithmetic*, Book II, chapter XXI, 2 defines analogia as in the proper sense, the combination of two or more ratios and it applies only to geometrical proportions. Hence analogia is the combination of symmetry and ratios. At times they are called marriage numbers, as a ratio of 7/50 can be expressed as $49/50 \times 1/7$, thus 49/50 is a marriage symmetry to bring the proportion to 1/7.

In Book I, chapter III, 2, it was pointed out that 'a building should have strength due to its foundation, utility due to its internal arrangements, and "grace" when the appearance of the work shall be pleasing and elegant, the scale of the constituent parts is justly calculated for symmetry'. Thus symmetry must be present in all details of the constituents elements of a building to make it graceful.

The same relation is expressed in Book I, chapter III, 3–4; 'proportion implies a graceful semblance; the suitable display of details in their context. This is attained when the detail of the work are of a height suitable to their breadth and of a breadth suitable to their length; in a word, when everything has symmetrical correspondence'.

'Symmetry also is the appropriate harmony arising out of the details of work itself; the correspondence of each given detail among the separate details to the form of the design of the design as a whole'.

In the chapter 'On the training of the architects' Vitruvius warns that 'the difficult problems of symmetry are solved by geometrical rules and methods', so the architect should study well the rules of geometry.

Thus symmetry combined with proportion was the harmony of a building. Each temple had its proper symmetry. Symmetry is a geometrical concept in its difficult forms. Thus when the relation of dimensions of the parts of a building is found, this relation is an analogia, a combination of symmetry and the proportion applied for the parts in question.

Chapter X in Euclid's *Elements* deals with symmetry. Symmetry is considered as the ratio between two unequal lines, if the given magnitudes have a common measure (whatever the measure may be). Thus, two lines one of five measures and the other of six measures, have a symmetry of 5/6, whatever the measure is.

The definitions preceding chapter X in Euclid's *Elements* define two kinds of symmetry. Symmetry is translated there by Sir T. L. Heath as commensurability. I prefer to use the word symmetry when quoting.

1 'those magnitudes are said to be symmetrical which are measured by the same measure, and those asymmetric which cannot have any common measure'.
2 'Straight lines are symmetrical in square when the squares on them are measured by the same area, and asymmetric in square when the squares on them cannot possibly have any area as a common measure'.

The above expressed definitions reveal the fact that Euclid had in mind application of a geometric series. For example, if A and B are two magnitudes with a common measure, hence symmetrical, then the mean term between A and B is $\sqrt{A \times B}$, and the ratio of

$$\frac{A}{\sqrt{A \times B}} = \frac{\sqrt{A}}{\sqrt{B}}.$$

Thus square roots of A and B are considered symmetrical in square, because, in square it gives A and B, and they were assumed to be symmetrical. This is attributed to Theaetetus by Plato in the book dedicated to him, although the same relations are found in Egyptian monuments, showing that the idea of symmetry had a common use in Egypt and probably later was taken over by the Greeks[1].

A second mean proportional between A and $\sqrt{A \times B}$ will lead to a symmetry of $A^{\frac{1}{4}}/B^{\frac{1}{4}}$.

Thus they are symmetrical in biquadratic form. That is why mathematically, Sir T. L. Heath reached the following consideration concerning chapter X; '. . . Book X formed a repository of results to which could be referred problems which depended on the solution of certain types of equations, quadratic and biquadratic but reducible to quadratics'. This conclusion is taken from the point of view of a mathematician, and is correct mathematically, but from the point of view of applied geometry, chapter X is to instruct the architect about the lines he is using, whether they have a common measure or not.

Definitions 3 and 4 deal with rational and irrational lines and Euclid proposes that any line can be considered rational, and irrationality will be with respect to the measure assumed. The example given by Pappus[2] in the commentary on Book X of Euclid (p. 72) expresses the point mentioned above, 'for example, if there be taken, on the one hand, a square whose measure is eighteen square feet, and on the other hand, another square whose measure is eight square feet', the relation is $\sqrt{8}$ to $\sqrt{18}$. This relation can be expressed

$$\frac{\sqrt{8}}{\sqrt{9}} \times \frac{1}{\sqrt{2}} = \frac{2}{3};$$

in this form the common measure is $\sqrt{2}$ feet[3], the $\sqrt{8}/\sqrt{9}$ is the symmetry and $1/\sqrt{2}$ is the proportion, this relation is known as side and diagonals, given by Theon of Smyrna, affirming that these relations harmonize the cofigurations (J. Depuis, p. 71).

Theon of Smyrna, in his introduction to Music, states that all harmony is numerical, and harmonic relations are in consecutive numbers; the same is asserted by Nicomachus of Gerasa in Book I, chapter VI; 'It must needs be, then, that scientific number, being set over such things as these, should be harmoniously constituted, in accordance with itself; not by any other but by itself . . . for the most fundamental species in it are two, embracing the essence of quantity (the monad and dyad, the sameness and otherness, the fundamental cosmic force) . . . odd and even, and they are reciprocally woven into harmony with each other, inseparably and uniformly, by a wonderful and divine Nature . . .'.

Thus the symmetry to be harmonious had to be in the form N into N $\pm$ 1, connected with a proportion in harmonic intervals, superparticular or superpartient numbers. Thus the form of the temple will be partaking sameness and otherness by odd and even numbers of its symmetry.

In Book IV, chapter IV, Vitruvius states that 'the length of the temple is so arranged that the breadth is half the length', but it is not specified, if it will be half the length in square or in linear forms. It can be stated that 1 and 2 correspond to lower and upper notes, while $\sqrt{2}$ is the note mean proportional in between. In the classical period it was assumed that both ear and eye act in the same way for harmony, that is, in numerical relations forming a geometric proportion. The above assumption can be verified in music by the interval of a lima, 243 and 256 vibrations, as an interval detectable by human ear.

$$\frac{243}{256} = \frac{\sqrt{9}}{\sqrt{10}}, \ 242.86$$

hence 243 is a mean proportional between 230 and 256. Thus the ear is acting in geometric proportions for the differentiation of notes.

The division of intercolumnal spacings in the Vitruvian form of temples are the same for the side and the front except for the middle spacing in the front for a corinthian temple, and side spacing for a doric (or at times for Ionic temples). For this reason the symmetry of any classical monument can be found from the frontal distribution of its columnal intervals, or pilasters with columnal intervals, distances being taken from axis to axis.

[1] 'All Problems about Pyramids' published in *The Rhind Mathematical Papyrus*, Vol. I (Oberlin, 1927, pp. 96–99) (revolve around the SEKED) the name given for the symmetry of Vitruvius, e.g. Problem 56, If a pyramid is 250 cubits high and the side of its base 360 cubits long, what is its SEKED?.

The process is to take $\frac{1}{2}$ of 360; 180, and find the ratio of 250/180 in units of cubit. Thus the author is eliminating the factor of $\frac{1}{2}$ in the formula

$$\frac{N \pm 1}{N} \times \frac{1}{2}$$

and getting directly the symmetry.

Problem 59, If a pyramid is 8 cubits high and the side of its base 12 cubits long what is its Seked?

He advises finding the ratio of

$$\frac{8}{6} = \frac{4}{3};$$

our way is

$$\frac{8}{12} = \frac{4}{3} \times \frac{1}{2}.$$

[2] The commentary of Pappus on Book X of Euclid's *Elements*, William Thomson, Cambridge Harvard University Press, 1930, has survived in Arabic only, showing that the symmetry was present in early arabic constructions, as a dome is never built on a square, but on a rectangle, whose sides must reveal the symmetry applied.

The small altar of the Temple of Jupiter in Baalbek, reconstructed with its original blocks in the courtyard in front of the temple, has the following dimensions, measurements taken from centre of pilaster to centre of pilaster for the horizontal dimension; the height is the elevation above the podium; breadth 659, length 732, height 695,

$$\frac{659}{732} = \frac{9}{10}, \ \frac{659}{695} = \frac{\sqrt{9}}{\sqrt{10}}, \ \frac{695}{720} = \frac{\sqrt{9}}{\sqrt{10}}.$$

The so-called Monumental Altar, published by P. Collart, and P. Coupel, is a higher altar, added later inside the rectangular courtyard of the temple, to the east of the above mentioned small altar. It has the same relation of symmetry for its dimensions as above. 'Not analysed in the publication.). Length 1,642, breadth 1,553, height 1,590,

$$\frac{1{,}553}{1{,}642} = \frac{\sqrt{9}}{\sqrt{10}}, \ 1{,}557.7, \frac{1{,}590}{1{,}642} = \left(\frac{9}{10}\right)^{\frac{1}{4}}, \ 1{,}599.3.$$

[3] Siegler has published a drawing of a stone with squares of 42 mm. on page 49 in *Architecture of Kalabsha*.

Ten of 42 mm. is 42 cm. $42/\sqrt{2} = 29.698$ cm., which is the dimension of an ancient foot measurement in the Middle East.

The Peribole of Kalabsha and some of the elevations are planned with the symmetry of $50/49 = 5\sqrt{2}/7$, so if one dimension is 7 ft. the other will be 50 divisions of the 42 mm. line.

1.1. The Temple of Artemis (Front View).

For example, the front six columns of the peristyle of the temple of Artemis are placed on two sides of a middle interval of 456 cm., and two of 373 cm. apart—the total breadth B = 1,948 cm. If all spaces were of 373 cms. then the breadth would have been 1,865 cms. Hence the ratio of 1,865 to 1,948 is the symmetry used for the temple of Artemis.

$$\frac{1{,}865}{1{,}948} = \frac{\sqrt{11}}{\sqrt{12}},\ 1{,}947.92 \text{ or } \frac{373}{1{,}948} = \frac{\sqrt{11}}{\sqrt{12}} \times \frac{1}{5}$$

(FIG. 1, 1)

The same symmetry should govern the length to breadth, the length L is 10 × 373 = 3,730 cms.

$$\frac{1{,}948}{3{,}730} = \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{2}.$$

(FIG. 1, 2)

Vitruvian and non Vitruvian peristyle temples

The temples with peristyle columns having 2N-1 columns on the side, N being the number of columns in front, have their proportion 1/2, while non Vitruvian temples have $\sqrt{1/2}$ ratio. Thus the intercolumnal number of spaces will indicate the form of the factor 1/2. This is correct if the temple is not enlarged to include another chamber as the case of the Parthenon.

If the Artemis temple had been of Doric form, the axial distances would have been

$$1{,}948 \times \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{5} = 406.9,$$

while the two extreme intercolumnal distances would be 363.6 each. It is the symmetry formula which decides the spacing of columns in Doric and at times in Ionic form temples, for the side contracted spacings.

A fine example is the so-called Temple of the Athenians' at Delos. It is an emphyprostyle temple with six columns in front and at the back. The breadth of the podium is 1,106 and the length is 1,885 cm.

$$\frac{1{,}106}{1{,}885} = \frac{\sqrt{11}}{\sqrt{12}} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{4}},\ 1{,}105.17$$

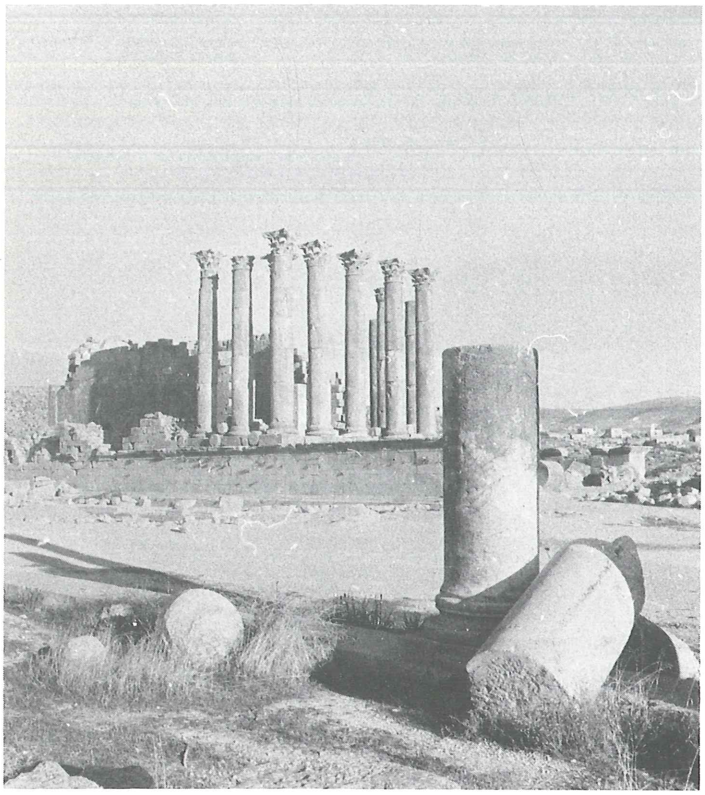
For the temple proper B is 879, L is 1,611.8 cm.

$$\frac{879}{1{,}611.8} = \frac{12}{11} \times \frac{1}{2},\ 879.16$$

Thus the mathematical evidence shows that the temple was for Athena or Artemis, because the symmetry is 12/11.

The frontal distribution of columns are 183.2 for the three

1.2. The Temple of Artemis (South View).



middle spaces, while the two side spaces are 164.7 cm. total 879 cm.

$$879 \times \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{5} = 183.6$$

instead of 183.2 cm.—an error of 0.4 cm.

It is difficult to ascertain whether the error is due to the architect reconstructing on paper or to the original planning, but both have done an admirable work of precision.

The same fact is true for the Parthenon, with this difference that in doric architecture, the middle intervals are uniform while the two last intervals are contracted, as in the example above.

The front peristyle intercolumnal distribution is: 5 × 429.9 + 2 × 365.5 = 2,880.5. If all were of 429.9 it would have been 7 × 429.9 = 3,009.3; the ratio of these two numbers is the symmetry of the temple

$$\frac{2{,}880.5}{3{,}009.5} = \frac{\sqrt{11}}{\sqrt{12}},\ 2{,}881.18$$

hence

$$2{,}881.18 \times \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{7} = 429.9.$$

The error in lay out, accounts for the differences found above, especially for some authors for whom the last spaces are 363.5.

The same symmetry should be for the breadth and the length; 14 × 429.9 + 2 × 265.5 = 6,749.6.

$$\frac{B}{L} = \frac{2{,}880.5}{6{,}749.6} = \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}},$$
$$2{,}878.0 \text{ or } 6{,}755.37.$$

To have a longer cella, with an adjoining chamber for treasury, the usual formula is multiplied by $1/\sqrt{3}$. All through, the symmetry of 11/12 is a factor in the relation of the parts to each other as well as to the whole.

For both cases, the Artemis and the Parthenon, it can be stated quite safely that the length was assumed as a whole number of feet; for Artemis the length will be 127 feet of 29.37 cm. each and such a measure is marked on the south side of the podium on a horizontal line; for the Parthenon the length is 228 feet of 29.59 cm.

The geometric method would be to describe a semicircle of 127 ft. diameter for Artemis and erect a perpendicular on the third division of the diameter of 127 ft. divided into 11 parts. The point found on the semi-circle joint to the end of diameter gives the B. In the case of the Parthenon, the diameter of 228 feet is divided into 11 parts and the perpendicular is from the second division.

$$\text{Artemis B} = \frac{127}{11} \times \sqrt{33} \quad \text{Parthenon B} = \frac{228}{11} \times \sqrt{22}$$

Such a method is well designed in the courtyard in front of the temple of Zeus in Jerash. It is assumed, from the inscription on the pediment that the temple of Zeus was completed in AD 163, the temple was to be placed on the high rocky ground to the south of the existing courtyard dated to c. AD 50, with its altar-like temple on its west side, with a small altar in front of it. This altar has been displaced, to give a full view to the stairs leading up to the temple of Zeus. This courtyard is surrounded with a vaulted corridor. The walls on the courtyard side have engaged columns with Ionic capitals at 300 cm. intervals; in between the columns there are alternating blind and open arches communicating with the corridor surrounding the courtyard. The diameter of the engaged columns is about 79 cm. and the opening of arches 143 cm. The symmetry of spacing to the diameter of the column is:

$$\frac{79}{300} = \frac{\sqrt{10}}{\sqrt{9}} \times \frac{1}{4} = 79.0 \qquad \text{(FIG. 2, 1\&2)}$$

Thus the spacing is four diameters, and the symmetry is 9/10. Hence the former temple was dedicated to a god equivalent to Zeus. The opening of arches are 2 diameters:

$$\frac{79}{143} = \frac{10}{9} \times \frac{1}{2} = 79.4.$$

The relation of the opening of arches to intercolumnal spacing is 2 as above:

$$\frac{143}{300} = \frac{\sqrt{9}}{\sqrt{10}} \times \frac{1}{2} = 142.3.$$

In this courtyard there are four concentric chiselled circles,

2.1. The Courtyard of the Temple of Zeus: the foundation of the displaced altar (centre); the inscription (slightly right of altar) and the chiselled circles on the courtyard slabs.

2.2. After restoration, the engaged columns with blind and open arches surrounding the courtyard of Zeus, with inscriptions AD 50.

the outermost one has a radius of $1{,}990 \pm 10$ cms. This dimension is a little over one half of the length of the temple of Zeus. On the east side of the temple the axis of the bases are marked with a cross, and the two intercolumnal distances is 714 cm. Hence the Length of the temple is $357 \times 11 = 3{,}927$ cm. This is the diameter of the second inner circle. The breadth of the temple is $2{,}640 \pm 5$ cms.

$$\frac{B}{L} = \frac{2{,}640}{3{,}927} = \frac{\sqrt{9}}{\sqrt{10}} \times \frac{1}{\sqrt{2}} = 2{,}634.3 \text{ cm.}$$

The temple is non-Vitruvian, with its 8 columns in front and 12 columns on the sides; thus the planning is done according to side and diagonal numbers as indicated by Theon of Smyrna.

One of the base stones of the displaced altar has an inscription in the form

IIIIINIIIII
IIIIINIIIII

which most probably indicates a measure of 120. The outer circle's diameter is 135 ft.; thus the measure used is one and one eighth of a foot (PL. 2, 1).

The statement by Vitruvius that the planning of temples depends on symmetry (and he is not stating what is the symmetry used) indicates that each god had its own symmetry. For houses Vitruvius gives three analogia with symmetries; (Bk. VI, Ch. V.III, 3),

$$\frac{6}{5} \times \frac{1}{2}, \quad \frac{4}{3} \times \frac{1}{2}, \quad \frac{\sqrt{2}}{1} \times \frac{1}{2}$$

and advises in the same chapter at 11; 'In buildings of this kind, all the rules of symmetry must be followed, which are allowed by the site, and the windows will be easily arranged, unless they are darkened by high walls opposite. But if they are obstructed by the narrowness of the street or by other inconveniences, skill and resource must alter the proportions by decreasing or adding, so that an elegance may be attained in harmony with the proper proportions'. Thus the advice is to keep the symmetry but change the proportion to have more light in case of obstruction of windows.

Having in mind that the numbers of symmetry give information about the function of buildings, and that on the Cardo of Jerash the presence of four columns with larger diameters in a compound, reveals the existence of an important building, these buildings were measured with the following results.

The building opposite to the parking ground in front of the present rest house, has four high columns spaced 383 cm., 430 cm. and 383 cm.; total 1,196 cm. This building is being excavated under the supervision of the Archeology Department of the Jordan University.

$$\frac{383}{1{,}196} = \frac{\sqrt{12}}{\sqrt{13}} \times \frac{1}{3} = 383.0 \qquad \text{(FIG. 3, 1)}$$

3.1. The building west of the Cardo, in front of the Rest House.

3.2. All that remains of the building which was on the parking ground.

This kind of symmetry is found in the temple of Bel in Palmyra. It might be called a symmetry of tolerance. On the outside of the entrance into the peribolus of the Temple of Bel, there are nine interpilasteral spaces on one side and ten on the other side, announcing a symmetry of 9/10. Immediately inside the peribolus, the portico columnade spacing on one side of a larger opening is 18 intercolumnal spaces, while on the other side it is 20 axial spacing, again a symmetry of 9/10. On the north and the south side of the courtyard the ratio of intercolumnal spacing numbers is $5/4 \times 1/2$; thus the symmetry is 5/4. While on the east side of the courtyard the ratio is 12/13. Thus the courtyard in its planning announces the presence of several gods with differing symmetries. All transformations in both thalamos were executed to bring the symmetries in line with the occupying god from the symmetry of 12/13 of the original planning of the temple. The stairs leading up to the temple were transformed into a podium by lowering the courtyard, to bring the symmetry into 9/10, because Bel was declared a Zeus or Jupiter. Thus the statement of Nonus addressed to the god of Tyre, that the gods are the same but their names are different, was a fact of symmetry attributed to gods long before the time of Nonus. It was a monotheism in polytheism.

The same symmetry is present at the main in antis entrance to the circular hall of the same building mentioned above. The distances are 346, 390, 346 total 1,082 cm.

$$\frac{346}{1{,}082} = \frac{\sqrt{12}}{\sqrt{13}} \times \frac{1}{3},\ 346.5.$$

The building which was on the parking ground, of which only 4 high front columns are standing, has the following spaces; 328, 422, 328; total 1,078 cm.

$$\frac{328}{1{,}078} = \frac{\sqrt{5}}{\sqrt{6}} \times \frac{1}{3},\ 328.0 \qquad \text{(FIG. 3, 2)}$$

According to the symmetries assigned to the non religious dwelling by Vitruvius, this building was either a residence or a Public building.

The next building on the west side of the Cardo, just before the entrance into the Cathedral, has 4 columns with intervals 351, 486, 351, total 1,188 cm.

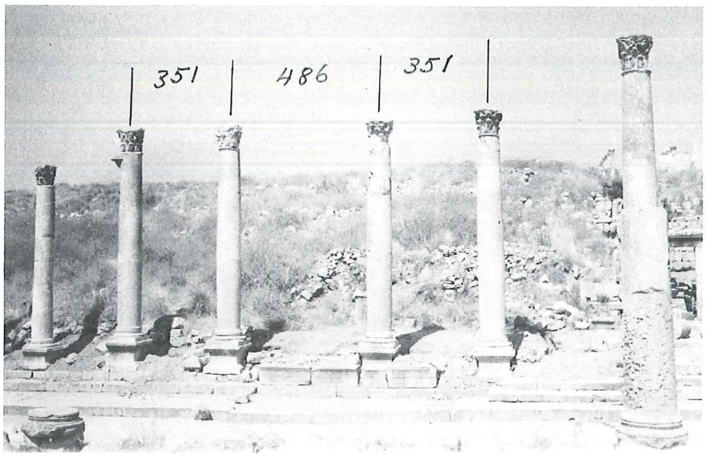
$$\frac{351}{1{,}188} = \frac{8}{9} \times \frac{1}{3},\ 351.99.$$

The 8/9 symmetry is found in Baalbek on an altar of pre-Roman construction when Baal of Baalbek was not a Jupiter Maximus, so this might be the temple of the Arabian god, to whom a dedication has been found (FIG. 4, 1).

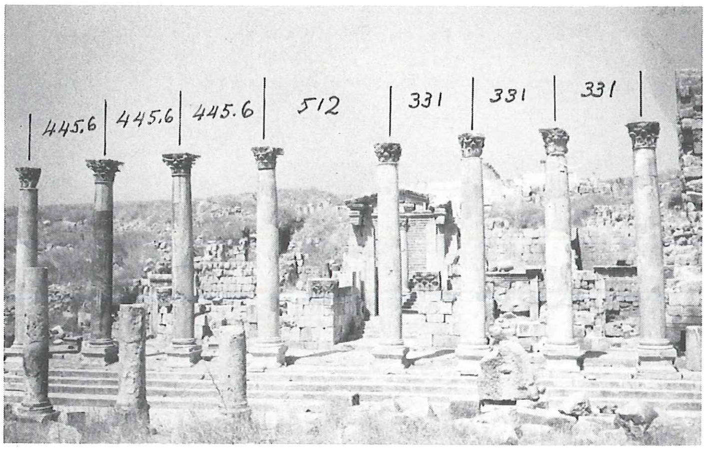
The entrance to the Cathedral has been left as it was in the pagan period, so its symmetry may reveal the gods worshipped in it. The columns have been restored on the bases as they were, but the pilasters in the background are in their respective positions. At present, the intercolumnal distances are, starting from the south; 412, 418, 397, 522, 331, 330, 334. The two pilasters flanking the entrance are 512 cm. apart. The distance from the southern pilaster to the pilaster on the south side of the door is 1,337 cm. so the distances of the southern intercolumnal spacing were 1,337/3 = 445.6 cm. On the northern side, the last column is displaced, the distances are 331 cm., totalling 993 cm., so the symmetry of the two sides is not the same. The symmetry of the southern side is (PL. 4, 2):

$$\frac{3{,}119.6}{3{,}186} = \frac{2 \times 1{,}337 + 445.6}{2 \times 1{,}337 + 512} = \frac{48}{49},\ 3{,}120.9$$

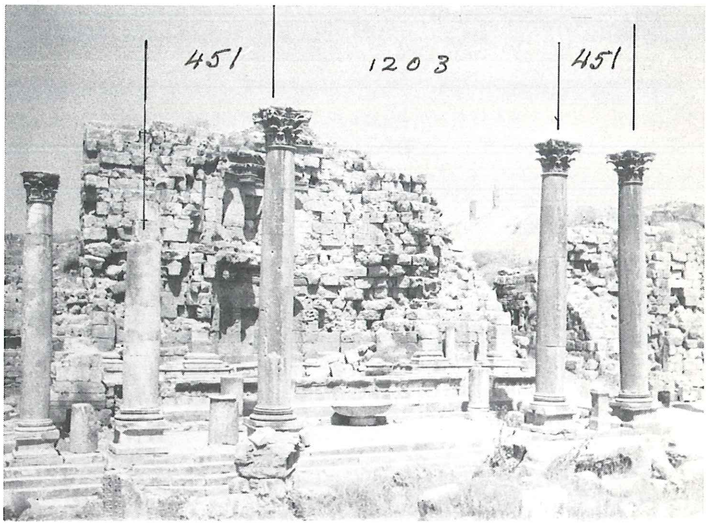
4.1. The four columns in front of the entrance to the Cathedral. (Temple of the Arabian God?)



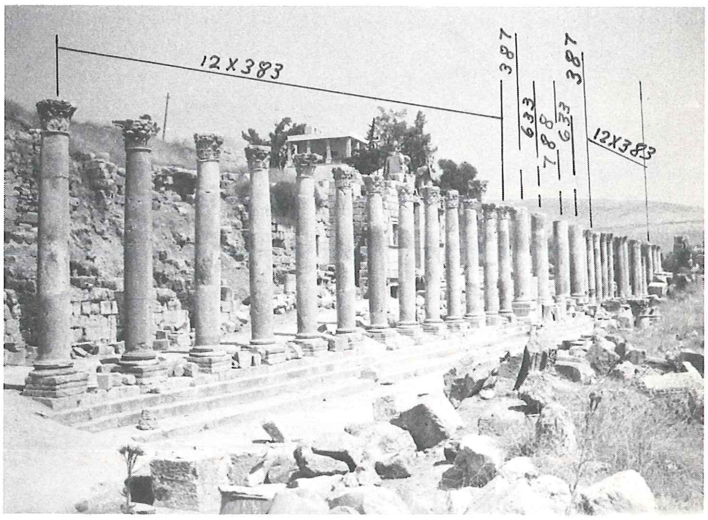
4.2. Entrance to the Cathedral.



5.1. Nymphaeum.



5.2. Entrance from the Cardo to the Artemis compound.



This is the symmetry of Bacchus, after the so-called temple of Bacchus in Baalbek (*Bulletin du Musée de Beyrouth*, T. XXIV, page 58, note 1). The presence of Bacchus was already inferred due to the presence of a historic miraculous fountain flowing wine on the day of Epiphany. Recent excavation in front of St Theodore, revealed the existence of a double water reservoir, connected with a lead pipe to the two fountains in front of St. Theodore's atrium. Most probably, the functioning of the smaller reservoir inside the larger one made the miracle possible.

The symmetry of the northern part is

$$\frac{2 \times 993 + 331}{2 \times 993 + 512} = \frac{12}{13}, \qquad \text{(FIG. 4, 2)}$$

Thus on the north side it has a tolerance symmetry, to allow for other gods being worshipped in the same precinct. At present there is the Cathedral and a monument to the Virgin Mary, and a passage towards the Temple of Artemis, known as the Sarapion passage, with an inscription on the lintel (no. 49) dated AD 65.

It is an ingenious design, having the same central opening for two different symmetries. The next building is the Nymphaeum with four columns; the distances are 451, 1,203, 451; total 2,105 cm.

$$\frac{451}{2,105} = \frac{\sqrt{5}}{\sqrt{6}} \times \frac{1}{\sqrt{2}} \times \frac{1}{3} = 452.9 \qquad \text{(FIG. 5, 1)}$$

This being the only example with 5/6 it is difficult to define the symmetry of the Nymphaeum. In the Lebanon, in Temnin el Fọqa the symmetry is 50/49.

The next building is the compound of Artemis, which has 29 intervals of columns; 12 on each side with 383 cm. spacing, next one of 387 cm. spacing, then 3 intervals with larger diameter of columns 633 cm., 788 cm., 633 cm. The sum of intervals is 12,020 cm. If all intervals were 383 cm., it would have been 29 × 383 = 11,107 cm.

$$\frac{11,107}{12,020} = \frac{12}{13}, \; 11,095 \qquad \text{(FIG. 5, 2)}$$

(The sum of the three middle larger spacing is 5 cm. less according to the wall pilasters opposing them.) The distribution of spacing of the columns has been mathematically as follows;

$$12{,}020 \times \frac{12}{13} \times \frac{1}{29} = 382.6 \qquad \text{(FIG. 4, 2)}$$

this has been for 12 intervals on each side, next two adjacent intervals

$$383 \times \frac{\sqrt{50}}{\sqrt{49}} = 386.88;$$

then the remaining space of 2,054 cm. is redistributed as;

$$2{,}054 \times \frac{12}{13} \times \frac{1}{3} = 632.$$

Thus, the outside boundary of the Artemis compound entrance was planned with a tolerance symmetry of 12/13 to include the statues of other gods on the cardo premises. At present there is evidence of three small chapels for statues.

The four free standing columns linked to the wall of the entrance doors, are spaced as 471 cm., 770 cm. and 479 cm. total 1,720 cm.

$$\frac{1{,}720}{2{,}054} = \frac{\sqrt{7}}{\sqrt{10}} = \left(\frac{49}{50} \times \frac{1}{2}\right)^{\frac{1}{4}},\ 1{,}718.5;$$

the middle spacing is a function of front spacing 788 × 49/50 = 772.2 the remaining space (1,720 − 772.2) is divided into two parts to make the spacing of columns on the side.

The three entrance doors on the Cardo, have the following measurements (the two side ones with slight differences), outside 267 by 418, inside 307 by 435, the middle one on the Cardo 505 by 925 cm.

The ratio of dimensions on two sides is:

$$\frac{267}{418} = \frac{\sqrt{11}}{\sqrt{12}} \times \frac{2}{3},\ 266.8$$

$$\frac{307}{435} = \frac{11}{12} \times \frac{20}{31}\left(\frac{20}{31} = \frac{30}{31}\right) \times \frac{2}{3}$$

$$\frac{505}{925} = \frac{12}{11} \times \frac{1}{2},\ 504.5.$$

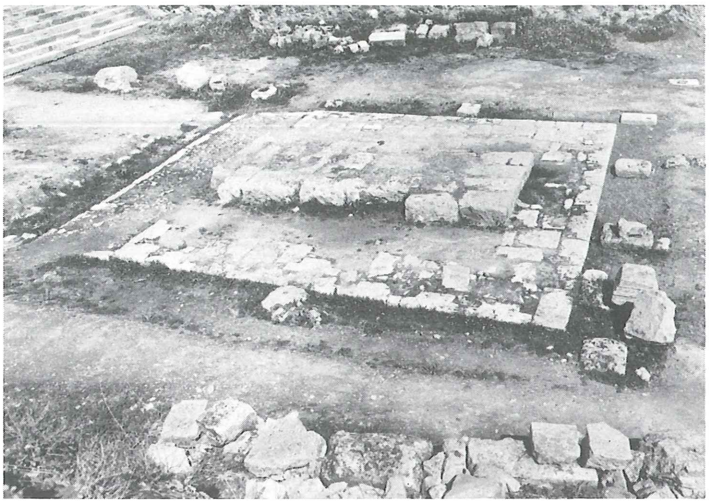
Thus the door closes the tolerance symmetry, and all statues inside the door should be of Artemis or her equivalent goddess.

The flight of 7 times 7 steps with plus one might well announce the end of the 49/50 marriage symmetry. On top of these steps there are foundations of an enlarged altar, the dimensions being 752 by 784 cm.

$$\frac{752}{784} = \frac{\sqrt{11}}{\sqrt{12}},\ 750.2.$$

The height of this altar must be in linear symmetry with the short dimension and in square with the long dimension. Thus the height of the altar was, assuming 784 is correct, 750.6 × 12/11 = 818.8 (FIG. 6, 1).

6.1. The Altar foundation in the Lower Court of the Temple of Artemis.



6.2. The Artemis Cella, the recess for the Statue and the segmented Arch.

This same relation is correct for both Altars in the courtyard of the Temple of Jupiter in Baalbek, the so-called Monumental Altar, and the small altar[4]. This fact is stated by Nicomachus of Gerassa and Theon of Smyrna. 'Solid figures in which the dimensions are everywhere unequal one to another . . . call the same numbers altars using their own metaphor, for altars of ancient style, particularly the Ionic, do not have the breadth equal to the depth, nor either of these equal to the length'. Theon of Smyrna calls small altars the cubes whose sides are not equal to each other.

The dimension of the courtyard's portico columnade, from centre to centre of engaged columns, is 9,150 by 12,385 cm.

$$\frac{9{,}150}{12{,}385} = \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{\sqrt{2}},\ 9{,}147.7.$$

In Kraeling, Fisher notes that the altar of the higher court, under the kiln, 'was not set on the axis of the temple but lay to the north of it, overlapping the axis by 80 cm.'. The stairs leading up to the Temple of Artemis are 1,720 cm.: the axis of the altar divides it into sections of 820 by 900 cm.

$$\frac{820}{900} = \frac{11}{12}$$

825 instead of 820, thus instead of 80 it should be 77 cm.

The lengths of the cella of the Temple, from centre of pilaster to centre of pilaster, are 2,624 by 1,206 cm.

$$\frac{1{,}206}{2{,}624} = \frac{11}{12} \times \frac{1}{2},\ 1{,}206.6.$$

The interior dimension of the Temple of Artemis at Epheseus is given as 71 ft. by 155 ft.

$$\frac{71}{155} = \frac{11}{12} \times \frac{1}{2},\ 71.04 \text{ ft.}$$

The interior dimension of the cella of the Temple of Artemis in Jerash is 1,100 by 1,816 cm.

$$\frac{1{,}100}{1{,}816} = \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{\sqrt{3}},\ 1{,}095.$$

The recess for the statue of Artemis is a rectangle of 395 by 430,

$$\frac{395}{430} = \frac{11}{12}.$$

This recess is covered by a segmented arch of 130 cm. high at the middle.

$$\frac{130}{395} = \frac{\sqrt{12}}{\sqrt{11}} \times \frac{1}{\sqrt{10}},\ 130.5. \qquad \text{(FIG. 4, 2)}$$

Thus all details of the temple had to obey the symmetry proper to the god. The question arises as to when and which international congress made these numerical decisions, and what did the numbers represent from a religious point of view.

During the pre-Christian periods all temples were planned according to the symmetry of the god concerned. All peristyle temples, having the number of side columns twice the number of the front columns minus one, are planned with the formula

$$\frac{\sqrt{N \pm 1}}{\sqrt{N}} \times \frac{1}{2}.$$

This fact is mentioned by Vitruvius, so they may be called Vitruvian temples. All other temples are planned with side and diagonal numbers, and they have one of the following forms

$$\frac{N \pm 1}{N} \times \frac{1}{2} \quad \left(\frac{N}{N \pm 1} \times \frac{1}{2}\right)^{\frac{1}{2}}$$
$$\left(\frac{N \pm 1}{N} \times \frac{1}{2}\right)^{\frac{1}{4}} \text{ or } \frac{N \pm 1}{N} \times \frac{1}{\sqrt{2}},$$

All these forms are consistent with each other on the supposition that $N \pm 1$ and N are two terms of a geometric series, as stated above.

When $N \pm 1$ and N are 10 and 9, the temple belongs to Jupiter or Zeus or an equivalent deity; when 12 and 11 it is Athena, Artemis, Atargatis, Cybele, perhaps Anahit or the equivalents; when 49 and 48 it is Bacchus Dusares or the equivalents; when 50 and 49 it is a human deity as Apollon, Hercules or deified Emperors.

Where there was a tolerance symmetry of 13 and 12 in a temple, *all* gods could be lodged, on condition that each lodge was arranged with the god's proper symmetry, the best example being the Temple of Bel in Palmyra.

In the Middle East 9 and 8 were attributed to gods who became later Jupiter Optimus Maximus, and took over the symmetry of 10 and 9, as at Baalbek. The reason for the change might have been that 9 and 8 in the form of side and diagonal have rational roots; as

$$\frac{\sqrt{9}}{\sqrt{8}} \times \frac{1}{\sqrt{2}} = \frac{3}{4} \text{ or } \frac{\sqrt{8}}{\sqrt{9}} \times \frac{1}{\sqrt{2}} = \frac{2}{3}.$$

Side and diagonal numbers with rational roots were cycles of the human race, as will be discussed below.

The Christian and Moslem periods

There is evidence, that the symmetry with its age-old allegiance to numbers has continued during the past Christian, Moslem and so-called pagan periods. One can state that the intrinsic attributes of the numbers transcended all religions.

There is one record of symmetry in Christian literature quoted by Crowfoot in Kraeling (Gerassa, p. 175) stating that 'the fore court (of the Baptistery) let it have . . . with its length 21 cubits for a type of total number of the prophets, and its breadth 12 cubits for a type of those who were appointed to preach the Gospel; one entry; three exits'. The ratio of 12 and 21 is

$$\frac{B}{L} = \frac{12}{21} = \frac{4}{7} = \frac{8}{7} \times \frac{1}{2},$$

that is why all baptisteries of later periods were planned in octagonal form, to emphasize the 8.

The church of St. John in Jerash measures 2,950 cm. by 2,380 according to Crowfoot;

$$\frac{2{,}380}{2{,}950} = \frac{8}{7} \times \frac{1}{\sqrt{2}},\ 2{,}383.9$$

error of 3.9 cm.

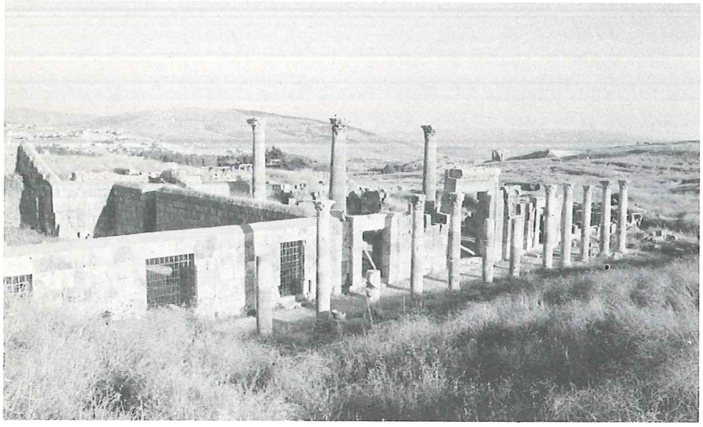
One example, with the same symmetry, but of pagan period is the chapel of Osiris (published in the Architecture volume of Kalabsha by Siegler), with dimensions 179 cm. by 334 cm.

$$\frac{179}{334} = \frac{\sqrt{8}}{\sqrt{7}} \times \frac{1}{2},\ 178.5$$

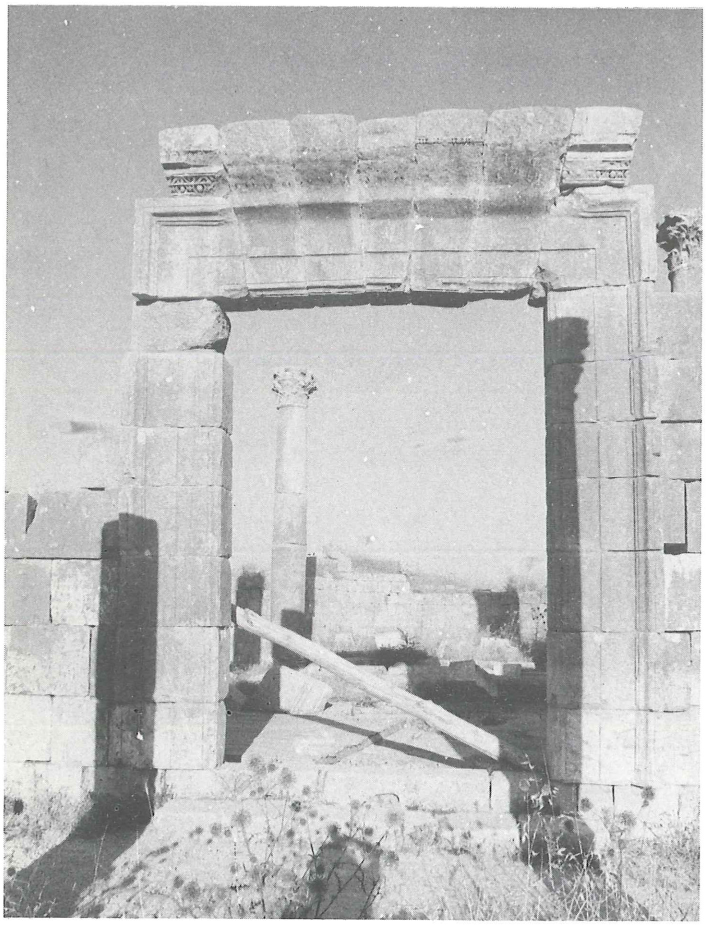
error 0.5 cm.

Other examples of earlier periods are the Pyramids, with

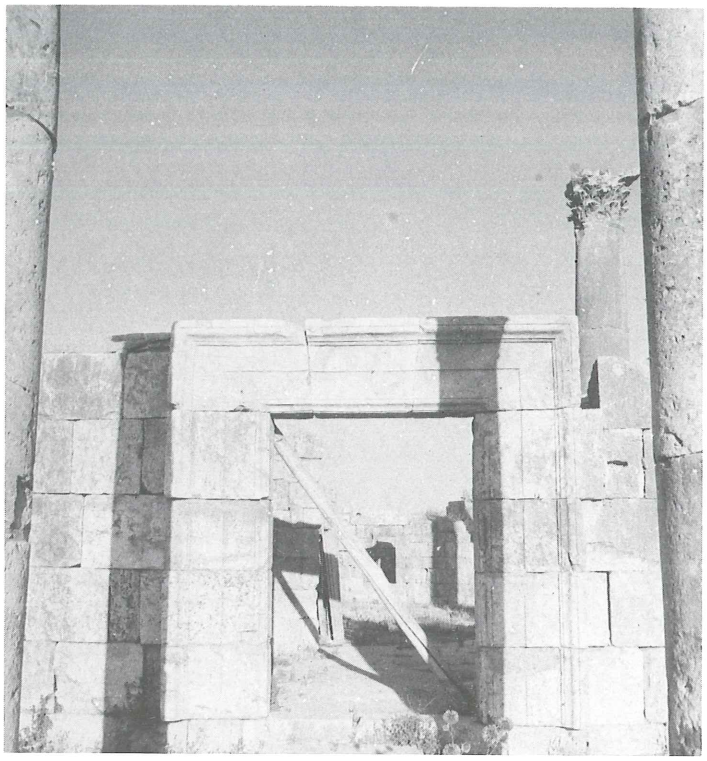
7.1. The restored entrance to the Church of St John.



7.2. Central entrance.



7.3. North entrance.



7.4. South entrance.

the assumption that these might have been, relative to dimensions involved, slight errors in approximations or extrapolations. (Dimensions are from I. E. S. Edwards[3].)

Zose-Ti pyramid base 395 ft. height probably 230 ft.

$$\frac{230}{395} = \frac{8}{7} \times \frac{1}{2}, 225.7$$

error 4.3 ft.

Bent pyramid, approximately 620 ft. at the base, height 336 ft.

$$\frac{336}{620} = \frac{\sqrt{8}}{\sqrt{7}} \times \frac{1}{2}$$

331.4 or 628.59 ft.

All these examples with 8 and 7 symmetry have a common denominator, that is heaven. St John and Osiris were sent from heaven, and the Pyramids were a communication channel with heaven. Thus persons or places related with heaven should have their monuments with a symmetry of 8 and 7 properly planned. The Church of the Resurrection and the Dome of the Rock in Jerusalem, due to their connections with heaven, had to be on an octagonal plan, with a symmetry of 8 and 7 and indeed they have been planned that way. (An article on this has been submitted for publication.)

The doors of the church of St John at Jerash were restored recently with the elements found on site. There are three doors (FIG. 7). The north door, opposite to the entrance into the baptistery, is 175 cm. wide and 253 cm. high.

$$\frac{175}{253} = \frac{\sqrt{49}}{\sqrt{50}} \times \frac{1}{\sqrt{2}} = \frac{7}{10},$$

177.1 error 2.1 cm.

With the side decorations (architraves) the outside dimension is 334 by 338

$$\frac{334}{338} = \frac{\sqrt{49}}{\sqrt{50}}, \ 334.6.$$

The central door is 273 cm. by 414 cm. with architraves it is 431 by 495 cm.

$$\frac{273}{414} = \frac{\sqrt{7}}{\sqrt{8}} \times \frac{1}{\sqrt{2}}, \ 273.8 \ \frac{431}{495} = \frac{7}{8}, \ 492.5.$$

The central door is constructed with elements taken from the temple of Zeus, such as the door jamb of the cella, with a symmetry of 10 and 9, and has been re-arranged to have the symmetry of 8 and 7.

The south door is 165 cm. by 263 cm.; with architraves the outside dimension is 324 by 330.

$$\frac{165}{263} = \frac{7}{8} \times \frac{1}{\sqrt{2}}, \ 162.7 \ \frac{324}{330} = \frac{49}{50}, \ 323.4.$$

The human cycle

The reference to the human cycle (symmetry) is in Plato's *Republic* viii 546 B-D (translation, Ivor Thomas, *Greek Mathematics*, Vol. I, p. 399) 'The divine race has a cycle comprehended by perfect number [*probably with no roots*] but the number of the human race's cycle is the first in which root and square increases, forming three intervals and four terms of elements that make like and unlike and wax and wane, show all things agreeable and rational towards one another. The base of these things, the four-three joined with five, when thrice increased furnishes two harmonies, the one a square, so many times a hundred, the other a rectangle, one of its sides being a hundred of the numbers from the rational diameters of five, each diminished by one (or a hundred of the numbers from the irrational diameters of five, each diminished by two, the other side being a hundred of the cubes of three'.

The base is four-three by five.

$$\frac{\sqrt{8}}{\sqrt{9}} \times \frac{1}{\sqrt{2}} = \frac{2}{3} \text{ added 5} \qquad \frac{\sqrt{9}}{\sqrt{8}} \times \frac{1}{\sqrt{2}} = \frac{3}{4} \text{ added 7}$$

$$\frac{\sqrt{50}}{\sqrt{49}} \times \frac{1}{\sqrt{2}} = \frac{5}{7} \text{ added 12} \qquad \frac{\sqrt{49}}{\sqrt{50}} \times \frac{1}{\sqrt{2}} = \frac{7}{10} \text{ added 17}$$

1st increase

$$\frac{\sqrt{288}}{\sqrt{289}} \times \frac{1}{\sqrt{2}} = \frac{12}{17} \text{ added 29} \qquad \frac{\sqrt{289}}{\sqrt{288}} \times \frac{1}{\sqrt{2}} = \frac{17}{24} \text{ added 41}$$

2nd increase

$$\frac{\sqrt{1{,}682}}{\sqrt{1{,}681}} \times \frac{1}{\sqrt{2}} = \frac{29}{41} \text{ added 70} \qquad \frac{\sqrt{1{,}681}}{\sqrt{1{,}682}} \times \frac{1}{\sqrt{2}} = \frac{41}{58} \text{ added 99}$$

Third increase

$$\frac{\sqrt{9{,}800}}{\sqrt{9{,}801}} \times \frac{1}{\sqrt{2}} = \frac{70}{99}$$

Thus the four 'term elements' are

$$\frac{5}{7} \quad \frac{12}{17} \quad \frac{29}{41} \quad \frac{70}{99}$$

with three intervals in between which are the years in which the human character is transformed. Cycle elements 5 to 7, and 12 to 17, are critical periods of formation; the same can be said for 29 to 41. Thus the formation of the roots with two numbers differing by one was considered as predestined for the human race.

Thus, for the human races the symmetry 50 and 49 was attributed and since this gave as roots 5 and 7 or 7 and 10 which could be used as well as the square roots of 5/7, this lead into the fourth root of 50/49 times 1/2.

The last section of Plato explains the square of 70 times 2 and the square of 99.

The mathematical analysis of the 'Eglise de Village de la Syrie du Nord', published by the Institut Francais d'Archeologie du Proche Orient (Paris 1979) gave the following symmetries for church construction.

The churches of Brad, Sergible, Kfeir are planned with $\sqrt{5}/\sqrt{4} \times 1/2$, Burg Heidar, Batuta, Kalota, Kafr Hawar, Qirqbize, Gerade with $\sqrt{7}/\sqrt{8} \times 1/\sqrt{2}$, Bafetin $\sqrt{8}/\sqrt{7} \times 1/\sqrt{2}$, Farfetin, Qalbloze, Kimar, Seih Sleiman $7/8 \times 1/2$, Betir $7/8 \times 1/\sqrt{2}$. These are all 8 and 7 symmetry so they must have been dedicated to Christ, St John, the Virgin Mary, the Redeemer or any heavenly dedication.

The churches of Kfeir Dartazze, Babisqa, Darqita, are planned with $6/5 \times 1/2$, Kala'at Kolata $\sqrt{5}/\sqrt{6} \times 1/\sqrt{2}$. 5 and 6 was the symmetry of the houses as stated by Vitruvius, so these churches were probably dedicated to local saints, groups of Apostles, or groups of saints.

The churches of Baqirha $\sqrt{6}/\sqrt{7} \times 1/\sqrt{2}$, Behyo $6/7 \times 1/\sqrt{2}$, Ruweiha South $6/7 \times 1/2$, are dedicated to an Apostle.

For human dedication we have St George of Bosra, the rectangle, 49/50, Sinhar, Harab Sams, $\sqrt{50}/\sqrt{49} \times 1/2$, Sugane, Gubelle, Kfellusin, Baduda, Dehes 7/10, Kafr Nabo 50/49 × 1/2, Bizzos 49/50 × 1/2

$$\text{Zebed} = \frac{\sqrt{7}}{\sqrt{5}} \times \frac{1}{2} = \left(\frac{49}{50} \times \frac{1}{2}\right)^{\frac{1}{4}} \times \frac{1}{\sqrt{2}}\,.$$

The symmetry of each church governs both its longitudinal section of aisles and naves, and the transverse section for the distances of the apse, and of the Bema, the proportion of Bema, as well as the vertical divisions of the facade elevation and the height.

This system of symmetry continued throughout the Gothic period and was lost with the Renaissance[4].

[4] Some examples from the Armenian churches covering the period from the IV to XI century.

Ererouk IV C. $\frac{2{,}363}{3{,}585} = \frac{\sqrt{7}}{\sqrt{8}} \times \frac{1}{\sqrt{2}}$, 2,371.2

EARLY Echmiatsin (estimated dimensions) $\frac{2{,}302}{2{,}450} = \frac{\sqrt{7}}{\sqrt{8}}$, 2,291.6

Ashtarak V–VI C. $\frac{1{,}165}{2{,}264} = \frac{50}{49} \times \frac{1}{2}$, 1,155.

Hripsime VII C. $\frac{1{,}769}{2{,}287} = \frac{\sqrt{6}}{\sqrt{5}} \times \frac{1}{\sqrt{2}}$, 1,771.5

Mren VII C. $\frac{2{,}000}{2{,}995} = \frac{\sqrt{7}}{\sqrt{8}} \times \frac{1}{\sqrt{2}}$, 1,981

Talish VII C. $\frac{1{,}878}{3{,}639} = \frac{50}{49} \times \frac{1}{2}$, 1,856 or 3,680

Marmashen IX C. $\frac{1{,}225}{1{,}950} = \frac{7}{8} \times \frac{1}{\sqrt{2}}$, 1,206.5 or 1,979.8

Horomos X C. $\frac{822}{1{,}515} = \frac{\sqrt{7}}{\sqrt{6}} \times \frac{1}{2}$, 818.19 or 1,522.

Ani Cathedral X C. $\frac{2{,}187}{3{,}429} = \frac{\sqrt{4}}{\sqrt{5}} \times \frac{1}{\sqrt{2}}$, 2,168 or 3,457.

Ani Apostles XI C. $\frac{1{,}953}{2{,}142} = \frac{\sqrt{5}}{\sqrt{6}}$, 1,955.3.